

A Parallel Novelty Search Metaheuristic Applied to a Wildfire Prediction System

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Jan Strappa^{ab}, Paola Caymes Scutari^{ab} and Germán Bianchini^a

^a Laboratory of Research in Distributed/Parallel Computing (LICPaD),
Systems Engineering Dept. - National Technological University - Mendoza - Argentina.

^b National Scientific and Technical Research Council (CONICET)

June 3, 2022

The wildfire propagation problem

Wildfires

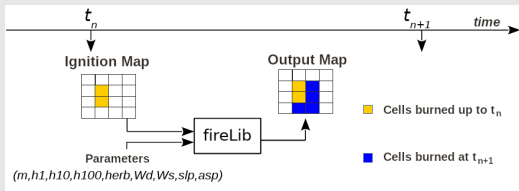
- ▶ Uncontrolled fires of rapid spread that affect land with vegetation such as forests, plains, grasslands, pastures, among others.
- ▶ Loss of human lives, evacuations, damage to flora and fauna, atmospheric emissions, soil erosion, major economic impacts.
- ▶ Millions of hectares per year burn globally; fire disasters more likely every year



Wildfire in Chubut, Argentina (March, 2021).

Prediction problem

Fire Simulator

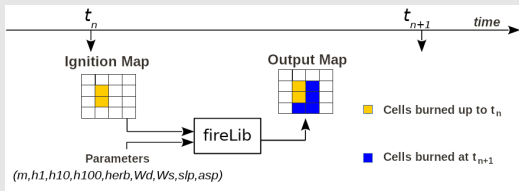


Input: scenarios (parameter vectors with environmental information)

Output: labeled map

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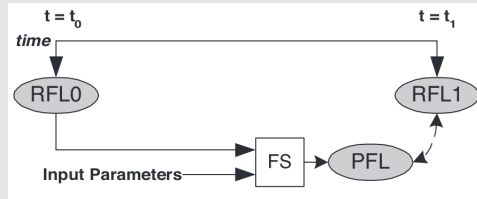
Fire Simulator



Input: **scenarios** (parameter vectors with environmental information)

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Classic Prediction



RFL0, RFL1: real fire line at times t_0, t_1

PFL: predicted fire line

Input parameters: scenario

FS: fire simulator

Uncertainty

Parameter	Description	Range	Unit of measurement
Model	Rothermel Fuel Model	1-13	fuel model
WindSpd	Wind speed	0-80	miles/hour
WindDir	Wind direction	0-360	degrees clockwise from North
M1	Dead Fuel Moisture in 1 hour since start of fire	1-60	percent
M10	Dead Fuel Moisture in 10 h	1-60	percent
M100	Dead Fuel Moisture in 100 h	1-60	percent
Mherb	Live herbaceous fuel moisture	30-300	percent
Slope	Surface slope	0-81	degrees
Aspect	Direction of the surface faces	0-360	degrees clockwise from north

Sources of uncertainty in input parameters

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- ▶ dynamically changing variables
- ▶ unknown values

Prediction methods and uncertainty reduction

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 - ▶ *ESS (Evolutionary Statistical System)*: optimization with metaheuristics, M/W
 - ▶ *ESSIM (Evolutionary Statistical System with Island Model)*:
double M/W hierarchy (Monitor → Masters of Islands → Workers).
 - **ESSIM-EA**: evolutionary algorithm (GA)
 - **ESSIM-DE**: differential evolution

Predecessors: ESS

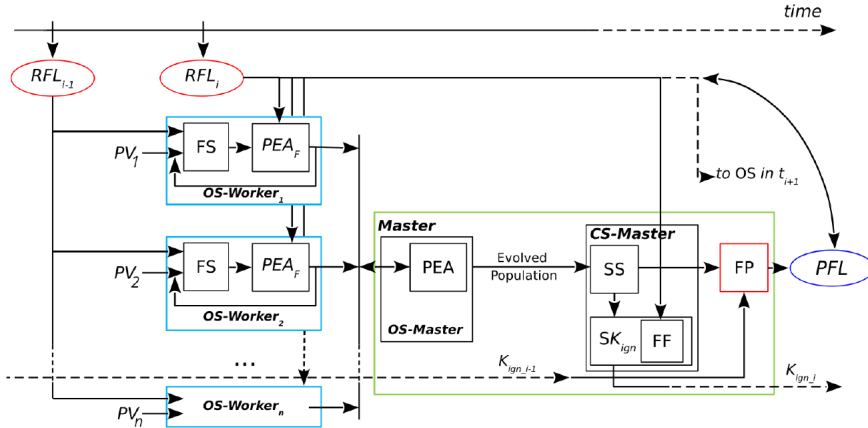


Fig. 1. Evolutionary Statistical System. *FS*: fire simulator, *OS-Master*: Optimization Stage in Master, *OS-Worker_x*: Optimization Stage in Worker x , *PEA*: Parallel Evolutionary Algorithms, *PEA_F*: Parallel Evolutionary Algorithm (fitness evaluation), *CS-Master*: Calibration Stage in Master, *FF*: fitness function, *RFL_i*: real fire line of instant t_i , *PFL_i*: predicted fire line of instant t_i , $PV_{\{1 \dots n\}}$: parameter vectors (scenarios), t_n : time instant n , K_{ign} : Key Ignition Value for t_i , *FP*: Fire Prediction stage, *SS*: Statistical Stage, *SK_{ign}*: Key Ignition Value search.

PEA_F computes the fitness $f(x)$ by comparing simulated vs real map (with Jaccard's index)

Previous methods: ESSIM-EA y ESSIM-DE

Main contributions of existing systems

- ▶ ESSIM-EA: improved quality and execution times than its predecessors
- ▶ ESSIM-DE: shorter execution times than ESSIM-EA, reaches acceptable quality fast but converges prematurely

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- ▶ ESSIM-DE presents premature convergence, quality improvements only with dynamic tuning techniques for diversifying results through randomness.

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A proposal that may overcome these limitations is to use the **Novelty Search** paradigm.

Novelty Search

Lehman, Joel & Stanley, Kenneth. (2008). Exploiting open-endedness to solve problems through the search for novelty. *Artificial Life – ALIFE*.

Motivation

In highly complex search problems, the **objective function** (in this case, the *fitness function*) is not enough to reach a solution, and can even prevent the search from doing so.

Consequences:

- ▶ premature convergence
- ▶ local optima

Novelty-based paradigm

Guide the search by the **novelty** of solutions instead of guiding it by the **objective function**.

Novelty Search approach

Description

- Define a **behavioral characterization** for the individuals (e.g., *fitness difference*).

genotype

$x = (m, h1, h10, h100, herb, Wd, Ws, slp, asp)$

(1, 40, 30, 10, 57, 45, 20, 65, 18)

phenotype

map simulated from x



fitness

$f(x)$

0.87

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- A **novelty score** function is computed for each new individual based on its behavior (w.r.t. others).
- The most novel individuals are selected:
 - for **population replacement**;
 - for inclusion in an **archive of past novel solutions**.
- The solution(s) of highest fitness are registered during the search.

Novelty Search approach

Novelty score function (*sparseness*)

The novelty of an individual is computed w.r.t. other individuals from the **current population** and the **archive of past novel individuals**.

$$\rho(x) = \frac{1}{k} \sum_{i=0}^{k-1} \text{dist}(x, \mu_i)$$

dist: measure of behavioral difference between two individuals in the search space.

k: number of closest neighbors (w.r.t. *dist*) to consider in the score.

μ_i : *i*-th closest neighbor of *x*.

Advantages of using *Novelty Search*

Advantages

- ▶ Avoids *by design* issues such as premature convergence (maximizes exploration).
- ▶ Easy implementation by modifying an existing metaheuristic:
 - ▶ replacing objective function by a measure of novelty;
 - ▶ adding an archive.

Proposal: Framework for *ESS - Novelty Search*

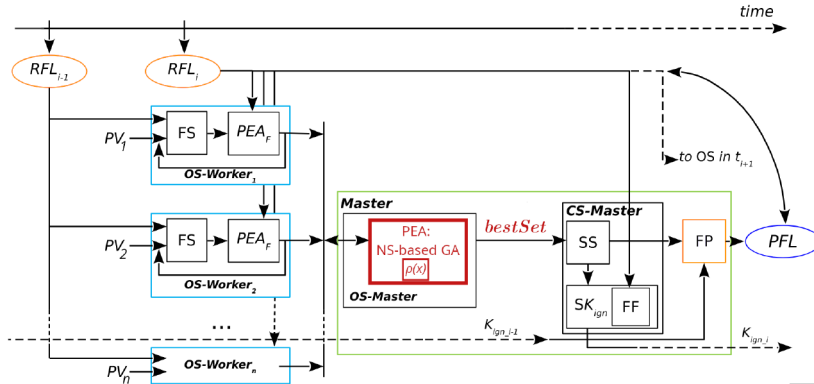


Fig. 3. Evolutionary Statistical System - Novelty Search. *FS*: fire simulator, *OS-Master*: Optimization Stage in Master, *OS-Worker_x*: Optimization Stage in Worker *x*, *PEA*: Parallel Evolutionary Algorithm, *NS-based GA*: Novelty Search-based Genetic Algorithm, $\rho(x)$: novelty score function (1), *PEA_F*: Parallel Evolutionary Algorithm (fitness evaluation), *CS-Master*: Calibration Stage in Master, *FF*: fitness function, *RFL_i*: real fire line of instant *t_i*, *PFL_i*: predicted fire line of instant *t_i*, *PV_{1...n}*: parameter vectors (scenarios), *t_n*: time instant *n*, *K_{ign,j-1}*: Key Ignition Value for *t_i*, *FP*: Fire Prediction stage, *SS*: Statistical Stage, *SK_{ign}*: Key Ignition Value search.

$$\rho(x) = \frac{1}{k} \sum_{i=0}^{k-1} f(x) - f(\mu_i) \quad (f(x) \text{ is computed at } PEA_F \text{ by workers})$$

Conclusions and future work

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- ▶ Preliminary results suggest NS obtained reasonable quality (in one case comparable to the best methods), and execution times comparable to the most efficient method.
- ▶ Promising results with straight-forward implementation, with many possibilities for improvement.
- ▶ This approach can be applied and adapted to other kinds of propagation problem.

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- ▶ This approach can be applied and adapted to other kinds of propagation problem.

Future work

- ▶ Use a different behavior characterization, e.g., simulated map.
- ▶ Improvements based on parameter calibration (k , archive & result set size), underlying metaheuristic, management of archive, etc.
- ▶ Algorithm variants with different criteria for the aggregation of results (not just fitness).
- ▶ Explore: parallelization, e.g., with an island model; hybridization with fitness-based search.

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Thank you!

Questions?

jstrappa@frm.utn.edu.ar

Additional slides

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3: bestSet  $\leftarrow$   $\emptyset$ 
4: generations  $\leftarrow$  0
5: maxFitness  $\leftarrow$  0
6: while generations < maxGen and maxFitness < fThreshold do
7:   offspring  $\leftarrow$  generateOffspring(population, m, mR, cR)
8:   for each individual ind  $\in$  (population  $\cup$  offspring) do
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<i>N</i>	population size
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12:  for each individual ind  $\in$  (population  $\cup$  offspring) do
13:    ind.novelty  $\leftarrow$  evaluateNovelty(ind, noveltySet, k)
14:  end for
15:  archive  $\leftarrow$  updateArchive(archive, offspring)
16:  population  $\leftarrow$  replaceByNovelty(population, offspring, N)
17:  bestSet  $\leftarrow$  updateBest(bestSet, offspring)
18:  maxFitness  $\leftarrow$  getMaxFitness(bestSet)
19:  generations  $\leftarrow$  generations + 1
20: end while
21: return bestSet

```

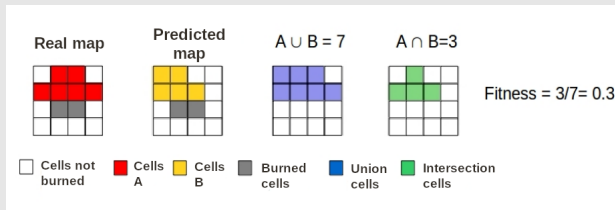
INPUT	
<i>N</i>	population size
<i>m</i>	# of descendants
<i>mR</i>	mutation rate
<i>r</i>	tournament prob.
<i>k</i>	neighbors (for $\rho(x)$)
<i>maxGen</i>	max # of generations
<i>fThreshold</i>	fitness threshold
OUTPUT	
<i>bestSet</i>	solutions

Predecessors: ESS, ESSIM-EA and ESSIM-DE

Optimization

Fitness function computation for each simulation uses **Jaccard's Index**:

$$fitness(A, B) = \frac{A \cap B}{A \cup B}$$



Predecessors: ESSIM-EA and ESSIM-DE

